Chapter 4. Word Problems

When arithmetic problems are expressed in words, complications arise because English is complicated and what is being requested in the problem is subject to interpretation. Modern students are generally not avid readers, and spend much time looking at graphics on video monitors. In algebra courses, solving a word problem requires two steps: (1) Translate the English into an algebraic statement of the problem and then, (2) Solve the algebra to obtain a solution. This can be symbolized as: **words → algebra → solution**. Often, in algebra classes, word problems are not covered because the chapter dealing with word problems is near the end of the text. Many students enter the university and enroll in general chemistry where they are expected to learn to solve word problems that deal with unfamiliar chemical terms. If they bring to the course little or no experience in solving word problems which deal with everyday terms and quantities, having to learn the chemical terms and dimensions makes chemical problem-solving a daunting task. In later chapters and in general chemistry, chemical problem-solving will be demonstrated. In this chapter, we will treat word problems that deal mostly with everyday quantities. We will attempt to be systematic and methodical in our approach, but our thinking will be different from the two-step procedure mentioned above. Our two-step approach will be as follows: (1) Reduce the verbiage by interpreting the problem as a unit conversion and/or a dimensional conversion problem, and (2) Solve the problem (*i.e.,* obtain the solution) by utilizing the unity-factor method. This approach will enable us to avoid being confused by wordiness in the problem statement. We will demonstrate the method by presenting a series of examples.

Example #1: Jack intends to spend three days driving to San Francisco, with a stop in Las Vegas. He intends to drive 550 miles the first day and 450 miles the second day, which will get him to Las Vegas. After a night's rest, he plans a day and evening of gambling and festivities, then a second night of rest. Then he intends to drive the last 250 miles to complete his journey. Since much of the trip involves driving on interstate highways, Jack feels like he can average 55 miles per hour while he is driving. How much actual driving time will the trip require if his estimated average speed is correct?

Solution. First, notice how many non-essential words are included in the problem statement. Knowing Jack's itinerary and his Vegas plans may be interesting, but they don't help us generate a solution to the problem. We need to look for a unity conversion factor, *i.e.*, a number having fractional units (a numerator and a denominator). In this case, it's the average velocity, 55 miles/hour. We can determine the total distance that Jack will drive in three days by adding the distances for each day of driving. Perhaps we should alter Jack's proposed distances by writing them to a reasonable number of significant figures.

5.5×10^2 miles + 4.5×10^2 miles + 2.5×10^2 miles = $(5.5 + 4.5 + 2.5) \times 10^2$ miles
= 12.5×10^2 miles.

Now we can use the unity dimensional conversion factor, the average velocity, to answer the question that was posed.

$$(12.5 \times 10^2\ \cancel{\text{mile}})\left(\frac{1\ \text{hr}}{55\ \cancel{\text{mile}}}\right) = 23\ \text{hr.}$$

Example #2. Jack had business to transact in San Francisco. He took with him \$25,000 to buy diamonds from a jewelry importer there. While in Las Vegas, Jack won at the blackjack tables and increased his money by 26%. If the diamonds to be purchased cost \$450 each, how many dozen diamonds can he purchase?

Solution. Again, there are superfluous words in this problem. While we may be interested to know that Jack likes to play blackjack, we need only know how much he added to his total amount of cash. We need a unity dimensional conversion factor to convert from dollars to quantity of diamonds. We have one, namely 450 \$/diamond. We can use it to convert from total money to number of diamonds purchased to the requested units, dozens. Here we go.

$$[25{,}000 + (0.26 \times 25{,}000)]\not{\$}\left(\frac{1\,\text{diamond}}{450\,\not{\$}}\right)\left(\frac{1\,\text{doz}}{12}\right) = 5.83 \text{ dozen diamonds.}$$

Note: Since .83 × 12 diamonds = 9.96 diamonds, he can purchase only 5 and 9/12 dozen diamonds = 5.75 dozen diamonds.

Thus, he will have some money left over to gamble with on his return trip through Las Vegas. How much money will be left over?

$$(5.83 - 5.75)\ \cancel{\text{dozen}}\ \cancel{\text{diamonds}}\left(\frac{12}{1\ \cancel{\text{dozen}}}\right)\left(\frac{450\ \$}{\cancel{\text{diamond}}}\right) = \$432.$$

Example #3. On his return trip, Jack encountered gas prices that averaged 1.63 \$/gallon. If his SUV averaged 20.3 miles per gallon, how much did gasoline cost to make the return trip home?

Solution. Obviously 1.63 \$/gallon and 20.3 miles/gallon are unity dimensional conversion factors. Let’s use them so that the units work this problem for us.

$$(12.5 \times 10^2\ \cancel{\text{mile}})\left(\frac{1\ \cancel{\text{gal}}}{20.3\ \cancel{\text{mile}}}\right)\left(\frac{1.63\ \$}{\cancel{\text{gal}}}\right) = \$100.$$

Example #4: Jack was away from home a total of 8 days and 15 hours. While he was away, a pipe in his basement sprung a leak and dripped water at a rate of 55 cm^3 per minute. If the leak started just as Jack left home, how many gallons of water dripped into his basement?

Solution: We have a time-to-volume unity conversion factor, namely, 55 cm^3/minute. Problem solution should be easy.

$$[(8 \times 24) + 15]\cancel{\text{hr}}\left(\frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}}\right)\left(\frac{55\ \cancel{\text{cm}^3}}{\cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{mL}}}{1\ \cancel{\text{cm}^3}}\right)\left(\frac{10^{-3}\ \cancel{\text{L}}}{1\ \cancel{\text{mL}}}\right)\left(\frac{1.057\ \cancel{\text{qt}}}{1\ \cancel{\text{L}}}\right)\left(\frac{1\text{ gal}}{4\ \cancel{\text{qt}}}\right) = 181 \text{ gal.}$$

Example #5: After fixing the leaky pipe and drying his basement, Jack decided to have a large dinner party. He decided to bake enough lemon icebox pies so that each guest could have one slice (a slice = 1/8 of a pie). The recipe calls for 1.33 cups of lemon juice per pie. An average lemon squeezes to produce 0.25 cups of juice. If Jack expects 50 dinner guests, how many lemons should he buy?

Solution:

$$(50\ \text{guests})\left(\frac{1\ \text{slice}}{\text{guest}}\right)\left(\frac{1\ \text{pie}}{8\ \text{slices}}\right)\left(\frac{1.33\ \text{cups of juice}}{\text{pie}}\right)\left(\frac{1\text{lemon}}{0.25\ \text{cups of juice}}\right) = 33\ \text{lemons.}$$

Example #6: In order to decorate for the dinner party, Jack bought a bunch of balloons and a cylinder of helium gas to fill them. He discovered that if he filled the balloons to a volume greater than 4.50 liters, they would pop. If the balloons were spherical, what is the largest diameter (expressed in inches) possible before a balloon pops?

Solution: This problem requires that we know a geometric formula. In this case, the formula needed is that for the volume of a sphere. You probably have memorized it to be $V = (4/3)\pi r^3$, where V= the volume, $\pi = 3.14286$, and r = the radius of the sphere. We can modify the equation to generate an expression for converting volume to length.

$V = \frac{4\pi}{3}\left(\frac{d}{2}\right)^3 = 0.5238\ d^3$, where d = the diameter of the sphere. Solving for d in terms of volume yields

$$d = \left(\frac{V}{0.5238}\right)^{1/3}.$$

$$d = \left(\left(\frac{4.50\ \text{L}}{0.5238}\right)\left(\frac{1\ \text{mL}}{10^{-3}\ \text{L}}\right)\left(\frac{1\ \text{cm}^3}{1\ \text{mL}}\right)\left(\frac{1\ \text{in}}{2.54\ \text{cm}}\right)^3\right)^{1/3} = 15.0\ \text{in.}$$

Example #7: At work, Jack was given the task of calculating how many ounces of gold would be required to gold-plate 100 spherical metallic beads, each having a radius of 2.00 cm, with a coating of gold 0.10 mm (= 0.010 cm) thick.

Solution: Jack reasoned as follows. The difference in the volume of a bead before and after it is coated equals the volume of gold required per bead. He could then use the density of gold to calculate the mass of gold needed per bead. Let's see if his plan works.

$$\left(\left(\frac{4\pi(2.01\,\text{cm})^3}{3}\right)-\left(\frac{4\pi(2.00\,\text{cm})^3}{3}\right)\right) = \left(\frac{4\pi}{3}\right)(8.12 - 8.00)\ \text{cm}^3 = 0.50\ \text{cm}^3/\text{bead.}$$

$$(100\ \text{beads})\left(\frac{0.50\ \text{cm}^3}{\text{bead}}\right)\left(\frac{19.32\ \text{g Au}}{\text{cm}^3}\right)\left(\frac{1\ \text{lb}}{453.59\ \text{g}}\right)\left(\frac{16\ \text{oz}}{1\ \text{lb}}\right) = 34\ \text{oz Au.}$$

Example #8: In addition to working at the jewelry store, Jack was a student at the local university. In his physics class, he learned that light from the moon takes 1.3 seconds to reach the earth. He was asked to use this information and the speed of light (1.86×10^5 miles/second) to calculate the distance in kilometers between the earth and the moon.

Solution: We recognize the speed of light to be the unity factor that enables Jack to convert from time to distance. He will have to use the speed to calculate the distance in miles, then convert from miles to km.

$$(1.3\ \cancel{s})\ (1.86 \times 10^5\ \frac{\cancel{mile}}{\cancel{s}})\left(\frac{1.6093\ km}{\cancel{mile}}\right) = 3.9 \times 10^5\ km.$$

Example #9: In Jack's chemistry class, he was introduced to a quantity known as a mole. 1 mole of anything equals 6.022×10^{23} of those things. He was a bit awed by the enormity of this number, especially after he did the problem assigned by his chemistry instructor: " If a mole of dollar bills, each 6.00 inches long were laid end-to-end at the earth's equator, how many times would the row of money go around the earth?"

Solution: At the equator, the earth's circumference is 39.3×10^3 km/circumnavigation.

$$(6.022 \times 10^{23}\ \cancel{dollars})\left(\frac{6.00\ \cancel{in}}{\cancel{dollar}}\right)\left(\frac{2.54\ \cancel{cm}}{1\ \cancel{in}}\right)\left(\frac{10^{-2}\ \cancel{m}}{1\ \cancel{cm}}\right)\left(\frac{1\ km}{10^3\ \cancel{m}}\right) = 9.18 \times 10^{19}\ km.$$

$$\left(\frac{9.18 \times 10^{19}\ \cancel{km}}{39.3 \times 10^3\ \cancel{km}/\text{circumnavigation}}\right) = 2.34 \times 10^{15}\ \text{circumnavigations.}$$

That's about two million billion, or two quadrillion times around the earth!

Example #10: It took Jack 4.3×10^9 microseconds to read and study this chapter. How much time is that in hours?

Solution: This example is simply a unit conversion within the dimension, time.

$$(4.3 \times 10^9\ \cancel{\mu s})\left(\frac{10^{-6}\ \cancel{s}}{1\ \cancel{\mu s}}\right)\left(\frac{1\ hr}{3600\ \cancel{s}}\right) = 1.2\ hr = 1\ \text{hour and}\ (0.2\ \cancel{hr})\left(\frac{60\ min}{1\ \cancel{hr}}\right) = 1\ \text{hour, 12 minutes.}$$

Of course, there are word problems that are not so easily solved by the reasoning demonstrated above. Some word problems require the assignment of x's and y's to stand for unknowns, followed by algebraic solution. Most of the chemistry problems that you will encounter in a general chemistry course can be solved by the unity-factor approach and will not require the words→algebra→solution approach.

The following web-references deal with the formula approach to solving word problems.
http://www.purplemath.com/modules/translat.htm
http://www.studygs.net/mathproblems.htm
http://www.purplemath.com/modules/numbprob.htm
http://www.edinformatics.com/math_science/scinot_mult_divc.htm
http://astro.temple.edu/~dhill001/wordproblemeqn/wordproblemeqn.html

Exercises

These exercises are more complicated than were those in the previous chapter. One complicating factor is the use of more words; another is the involvement of geometric formulas. These problems require more thought on the student's part and cannot be solved by rote adherence to procedure. Students should not be discouraged if they have more trouble with these problems than they had with those in the previous chapter. If you experience an inordinate amount of trouble, you might consider going on to the next chapter and returning to these problems at a later time, perhaps after completing the rest of the course.

For the following word problems, we will use the same dimensional conversion equations that were used in the previous chapter. They are reproduced here for convenience.

speed of light, c = 299,792,458 meters/second = 2.998×10^8 meters/second.
legal speed limit on two-lane highways in most states = 55 miles/hour [exactly].
density of a relatively light metal, aluminum (chemical symbol, Al) =
2.70 grams/milliliter.
density of a relatively heavy metal, lead (chemical symbol, Pb) = 11.34 grams/milliliter.
density of water (chemical symbol, H_2O) at $25^{O}C$ = 0.9970 grams/milliliter.
On March 30, 2005, at a convenience store in Conway, AR, the price of eggs was $1.20/dozen.
On March 30, 2005, the price of gold (Au) was $429.00 per troy ounce.
On March 30, 2005, the price of crude oil was $53.80 per barrel.

1. The distance from the earth to the moon is 3.845×10^5 kilometers. How long does it take (in seconds) for light leaving earth to bounce off the moon and return to its point of origin?

2. If a light beam bounced off a satellite orbiting the earth requires 0.15 seconds to return to earth, how many miles above the earth is the satellite?

3. If one drives a distance of 73.0 miles at a speed of 65.0 miles per hour rather than the legal speed limit, how many minutes of time will be saved?

4. If one drives for 3.00 hours at a speed of 65.0 miles/hour rather than at the legal speed limit, how much further (in kilometers) will he/she travel?

5. A sphere of aluminum, having a diameter of 4.00 inches, has what mass in ounces? Remember that the volume of a sphere is given by the formula, $V = \frac{4}{3}\pi r^3$, where r = the radius of the sphere, or half the diameter, and π = 3.14286.

6. A cube of lead has a mass of 100 pounds. What is the length of a side of the cube in inches? Remember that the volume of a cube is given by $V = s^3$, where s is the length of a side.

7. A reservoir initially holding 1.4×10^4 gallons of water, develops a leak causing 3.00 grams of water to be lost per minute. How many years will it take for the reservoir to be completely emptied, assuming that no water is being added. Assume that the water temperature is 25^{O}C. Also assume that the loss of water due to evaporation equals the gain of water due to rain.

8. If the leaking reservoir in problem 7 has water being added to it at the rate of 1.20 grams of water per minute, how many years will it take for the volume to drop to one-third of its initial value?

9. On 03/30/05, how many barrels of crude oil could be purchased with 15 (exactly) ingots of gold? Each ingot is 6.00 inches by 3.00 inches by 2.00 inches. The volume of an ingot is given by: $V = l \times w \times h$. Given: density of gold equals 19.32 grams/mL.

10. On 03/30/05, how many ounces of gold could be purchased for the price of 30 thousand barrels of crude oil. Useful information is given in the preceding problem.